EFFECT OF VELOCITY AND TEMPERATURE FLUCTUATIONS ON PITOT PROBE MEASUREMENTS IN COMPRESSIBLE FLOW

30 JANUARY 1962

NOL
UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

NOLTR 61-28

Aeroballistic Research Report No. 151

EFFECT OF VELOCITY AND TEMPERATURE FLUCTUATIONS ON PITOT PROBE MEASUREMENTS IN COMPRESSIBLE FLOW

by

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ABSTRACT: The effect of velocity and temperature fluctuations on the pressure indicated by a Pitot probe has been analyzed for the compressible case assuming negligible static pressure fluctuations. This analysis is based on the assumption that the Mach number fluctuation in the free stream ahead of the probe affect the Pitot pressure directly. As a result, the measured Pitot pressure divided by the static pressure is not just a function of the average Mach number and the ratio of specific heats as it is in steady flow. In order to correctly interpret Pitot pressure measurements in a turbulent boundary layer it is necessary to separate from the measurements the effects of the Mach number fluctuations.

The results of the analysis show that the velocity fluctuations directly and also via the temperature fluctuations, cause the indicated Pitot pressure to be greater than the Pitot pressure associated with the time average velocity and temperature. Velocity fluctuations cause an increase in the Pitot pressure in proportion to the mean of the square of the fluctuation as is already known for incompressible flow. The role of the temperature fluctuations is to increase the effect of the velocity fluctuations on the measured pressure. Heat transfer into the wall, acting through the temperature fluctuations, has a reverse effect.

PUBLISHED MARCH 1962

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

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Effect of Velocity and Temperature Fluctuations on Pitot Probe Measurements in Compressible Flow

This report is a theoretical analysis of the effect of fluctuating velocity and temperature (as in turbulence) on the measurement of Pitot pressure in compressible flow. This type of information has application in estimating possible measurement errors in experimental hypersonic turbulent boundary-layer research. In such research it is particularly important to measure the Pitot pressure accurately in the vicinity of the wall because such an important parameter as skin friction is sensitive to this measurement. The results of the analysis indicate that velocity fluctuations directly and also indirectly through the temperature fluctuations increase the Pitot pressure above the Pitot pressure associated with the time average velocity and temperature. Therefore, if accurate pressures are required, this effect must be considered.

This analysis was performed in connection with the experimental program of measuring the characteristics of the hypersonic turbulent boundary layer which is jointly sponsored by the Bureau of Naval Weapons under Task No. RMGA-42-034/212-1/F009-10-001 and Special Project Office, Bureau of Naval Weapons under Task No. PR-9.

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By direction

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SYMBOLS

A_n	coefficients defined in equation (18)
b	$\frac{\gamma - 1}{2} \bar{\bar{M}}^2$
C	velocity of sound
C_w	velocity of sound based on wall temperature
C_p	specific heat at constant pressure
M	Mach number
$\bar{\bar{M}}$	$U^2/\gamma R \left(\bar{\bar{T}}_o - \frac{U^2}{2C_p} \right)$
p	static pressure
p_o'	Pitot pressure
$\bar{\bar{p}}_o'$	Pitot pressure associated with $\bar{\bar{M}}$
R	gas constant
$R_{T'V'}$	correlation coefficient $= \frac{\sqrt{\overline{T'V'}}}{\sqrt{\overline{T'^2}} \sqrt{\overline{V'^2}}}$
T_o	total temperature
T_{o_∞}	free-stream total temperature
T_w	wall temperature
U	x-component velocity
U_∞	free-stream velocity
V	resultant velocity
x	co-ordinate parallel to the wall
y	co-ordinate normal to the wall
y^+	$\frac{\sqrt{\tau_w \rho_w}}{\mu_w} y$ non-dimensional wall distance

α	defined in table 1
β	$\dfrac{2C_p}{\bar{U}} \dfrac{d\bar{T}_o}{d\bar{U}}$
γ	ratio of specific heats = 1.4
δ	boundary-layer total thickness
ϵ	$\dfrac{U'}{\bar{U}}$
θ	angle between instantaneous velocity vector and U - component
μ_w	viscosity based on wall temperature
ρ_w	density based on wall conditions
τ_w	shear stress on the wall
$\overline{}$	a bar over any symbol indicates time average
$'$	a prime after any symbol indicates the fluctuating part of the quantity indicated by the symbol

Subscripts

m	maximum value
w	wall conditions
∞	free-stream conditions

INTRODUCTION

1. The direct measurement of local shear stress is difficult under any condition but particularly so in hypersonic turbulent boundary layers. The indirect method of obtaining skin friction from boundary-layer surveys appears simpler to perform experimentally than employing a skin-friction balance. For example, the wall shear stress can be calculated from

$$\tau_w = \mu_w C_w \left(\frac{\partial M}{\partial y}\right)_w \tag{1}$$

where μ_w = coefficient of viscosity based on T_w
C_w = velocity of sound based on T_w
and where $(\partial M/\partial y)_w$ is the Mach number gradient based on an extrapolation of the Mach number profile to the wall. The Mach number profile is obtained from Pitot probe surveys and the local static pressure. There are, however, questions concerning the interpretation of these probe measurements in a turbulent flow.

2. In steady laminar flow, the pressure indicated by a Pitot probe can be related to the local Mach number because the Pitot pressure uniquely depends on the square of the Mach number in the free stream ahead of the probe. For example, in compressible flow ($M \leq 1$)

$$P_o'/P = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1} \tag{2}$$

and in supersonic flow (Rayleigh Pitot Formula $M \geq 1$).

$$P_o'/P = \left[\frac{\gamma+1}{2} M^2\right]^{\gamma/\gamma-1} \left[\frac{\gamma+1}{2\gamma M^2 - (\gamma-1)}\right]^{1/\gamma-1} \tag{3}$$

3. A problem exists, however, when the flow is turbulent because the fluctuations in the free stream ahead of the probe affect the pressure indicated by the probe. Even close to the wall, in the laminar sublayer region, existing experimental data indicate that the fluctuations are large compared with the mean value. The following is an estimate of the effect these fluctuations have on the probe reading with particular attention to the region near the wall in a supersonic turbulent boundary layer.

ASSUMPTIONS CONCERNING THE AVERAGE PITOT PRESSURE RATIO

4. It is not obvious from the relation between the uniform flow Pitot pressure and free-stream Mach number (eqs. (2) and (3)) what type of relation exists between the time average Pitot pressure and the time average Mach number. If it is assumed that equations (2) and (3) hold instantaneously for the time dependent quantities, then a particularly simple relation is obtained if both equations (2) and (3) are approximated by:

$$P_o'/P \cong 1 + M^2 \tag{4}$$

This approximation predicts the uniform flow Pitot pressure within 13 percent in the region $0 \leq M \leq 2$ (see fig. 1). This range of Mach number is acceptable for the present purposes because the region of particular interest in the turbulent boundary layer is near the wall where the Mach number is two or less. The probe reading then is simply the time average of the instantaneous pressure (eq. (4)).

$$\overline{P_o'}/P = 1 + \overline{M^2} \tag{5}$$

5. Some justification can be given in the subsonic case for employing the time average of the approximation to equation (2). That is, if the time average is taken of the time dependent momentum equation along a streamline, written in terms of the total to static pressure ratio and Mach number, the result obtained is equation (5). This might be expected since, in the incompressible case as considered by Goldstein, the same kind of result is obtained (ref. (a)).

6. It is, however, much more difficult to show that equation (3) is valid in the presence of turbulence. Generally it is assumed that equation (3) does hold for slowly changing flow conditions. In this connection several theoretical papers (refs. (b), (c), and (d)) concerned with the changes produced in a shear wave as it passes through a shock wave start with the assumption that the steady state shock-wave relations (Rankine-Hugoniot relations) hold instantaneously for the flow variables as they pass through the shock waves. Inherent in this assumption are the limitations that the disturbances are small and that the rates of change of the flow variables are much "slower" than the speed with which the shock wave is able to adjust to the new conditions. This is an area where more experimental work is needed in order to qualitatively determine the limitations.

7. Therefore, in the absence of contradictory experimental or theoretical information equation (5) will be assumed in the following approximate calculations.

8. The average pressure in equation (5) depends on $\overline{M^2}$ and not on $\overline{M}^2$ so that if the instantaneous Mach number can be written as a mean value plus a fluctuating component, then $\overline{M^2}$ is larger than $\overline{M}^2$ by the amount $\overline{M'^2}$. The static pressure has been assumed constant through the boundary layer and independent of the fluctuations. A brief discussion justifying this assumption is given by Kistler in reference (e) in connection with the interpretation of hot-wire measurements. If $\overline{\overline{p_o'}}$ is defined as a Pitot pressure associated with a Mach number $\overline{\overline{M}}$ corresponding to the time average velocity and total temperature,

$$\overline{\overline{P_o'}}/P = 1 + \overline{\overline{M}}^2 = 1 + \frac{\overline{V}^2}{\gamma R(\overline{T_o} - \overline{V}^2/2C_p)} \tag{6}$$

then equations (5) and (6) can be combined to give:

$$\overline{\overline{P_o'}}/P = 1 + \left(\frac{\overline{P_o'}}{P} - 1\right)\frac{\overline{\overline{M}}^2}{\overline{M^2}} \tag{7}$$

The problem of finding $\overline{\overline{p_o'}}/p$ reduces to estimating $\overline{\overline{M}}^2/\overline{M^2}$.

FORMULATION OF $\overline{M^2}$ - RELATION

9. The instantaneous Mach number squared can be written:

$$M^2 = V^2/\gamma R T \tag{8}$$

where the instantaneous static temperature is:

$$T = T_o - V^2/2C_p \tag{9}$$

In this formula V is the magnitude of the resultant velocity vector which can be written:

$$U = V\cos\theta \tag{10}$$

If the cross components of the fluctuations are small then V is nearly equal to U, because the angle between V and U is small. When each of the variables in equations (8) and (9) is replaced by a mean value plus a fluctuating component; i.e.,

$$U = \bar{U} + U'$$
$$T_o = \bar{T}_o + T_o' \tag{11}$$

and when the square of the Mach number is formed from $\bar{U}$ and $\bar{T}_o$ as follows ($\cos\theta \approx 1$):

$$\bar{\bar{M}}^2 = \frac{\bar{U}^2}{\gamma R(\bar{T}_o - \bar{U}^2/2C_p)} \tag{12}$$

(Note that $\bar{\bar{M}}$ is not necessarily equal to the time average Mach number but is a Mach number associated with the time average T_o and U.)

then substitution of equations (9), (11), and (12) into equation (8) results in

$$M^2 = (\bar{U}+U')^2/\gamma R(\bar{T}_o + T_o' - (\bar{U}+U')^2/2C_p)$$

$$\frac{M^2}{\bar{\bar{M}}^2} = \frac{1 + 2\frac{U'}{\bar{U}} + \left(\frac{U'}{\bar{U}}\right)^2}{1 + \left(T_o' - \frac{\bar{U}^2}{2C_p}\left[2\frac{U'}{\bar{U}} + \left(\frac{U'}{\bar{U}}\right)^2\right]\right)\Big/(\bar{T}_o - \bar{U}^2/2C_p)} \tag{13}$$

10. The total temperature fluctuations can be approximated by considering a small element of volume in the boundary layer with properties T_{o1}, T_1, U_1 (element (1)) which is being displaced by a component in velocity normal to the mean streamlines into another region. In the new region just before the introduction of the fluid from element (1) the properties were T_{o2}, T_2, U_2. A stationary observer in the latter region observes fluctuations in each property, namely $U_2 - U_1$, $T_{o2} - T_{o1}$, $T_2 - T_1$. This suggests the possibility of relating the fluctuations in T and T_o to the fluctuations in U. If it is assumed that the average total temperature is just a function of the average velocity, a concept carried over from laminar boundary-layer theory; and if U' is a small velocity difference between the two regions of fluid, then the difference in T_o between these same two layers of fluid can be approximated by:

$$T_o' = \frac{d\bar{T}_o}{d\bar{U}}U' \tag{14}$$

Using the definitions:

$$\beta = \frac{2\,C_p}{\bar{U}}\,\frac{d\,\bar{T}_o}{d\,\bar{U}}$$

$$\eta = U'/\,\bar{U} \tag{15}$$

$$b = \frac{\gamma - 1}{2}\,\bar{\bar{M}}^2$$

equation (13) can be written:

$$M^2 = \bar{\bar{M}}^2\,\frac{\left[1 + 2\eta + \eta^2\right]}{\left[1 - b\left((2-\beta)\eta + \eta^2\right)\right]} \tag{16}$$

If the division in equation (16) is performed there results an infinite series of the form:

$$\frac{M^2}{\bar{\bar{M}}^2} = A_o + A_1\eta + A_2\eta^2 + A_3\eta^3 + \cdots A_n\eta^n + \cdots \tag{17}$$

where

$$\begin{aligned} A_o &= 1 \\ A_1 &= 2 + b(2-\beta) \\ A_2 &= 1 + b + b(2-\beta)(2 + b(2-\beta)) \\ A_3 &= b\,A_1 + b(2-\beta)A_2 \\ &\vdots \\ A_n &= b\,A_{n-2} + b(2-\beta)A_{n-1} \qquad , n \geq 3 \end{aligned} \tag{18}$$

equation (17) converges for $\bar{\bar{M}} < 2$ if $\beta = 0$ and $M' < 1.0$ (see Appendix A). When $\beta > 0$ i.e., heat transfer into the wall, the range of convergence is increased. In order to obtain numerical results it is assumed that M' and thus η are so small that terms in equation (17) of higher order than $A_2\eta^2$ are small. If then, the time average is taken of equation (17) one obtains the result:

$$\frac{\overline{M^2}}{\bar{\bar{M}}^2} = 1 + \left[1 + b + b(2-\beta)(2 + b(2-\beta))\right]\overline{\eta^2} \tag{19}$$

Equation (19) can be used in conjunction with equation (7) to estimate the Pitot pressure ratio correspond to the average flow conditions ($\bar{T}_o$ and $\bar{U}$) if $\overline{\eta^2} = \overline{(U'/\bar{U})^2}$ is known. An iterative procedure is required since $\bar{\bar{M}}^2$ must be first estimated in order to calculate b, but fortunately, the iteration converages rapidly. An estimate based on the measured Pitot pressure ratio ($\bar{p}_o'/p$) is a good first choice.

11. In order to complete the calculation, values of $\overline{\eta^2}$ are required for the compressible cases. The available compressible-boundary-layer information on this quantity (refs. (e), (f), and (g)) is limited. Measurements closest to the wall are at a $y^+ = 80$. The magnitude of $\overline{(U'/U)^2}$ is, in general, within the scatter of the incompressible data, although Kistler's data (ref. (e)) does indicate a decrease of $\overline{(U'/U)^2}$ with increasing Mach number. Because of this, it appears reasonable and conservative to employ the incompressible data (fig. 2) near the laminar sublayer where similarity in the u^+ versus y^+ profile is expected even in the compressible case.

COMPARISON WITH HOT-WIRE EXPERIMENTS

12. Before calculating the effect of the velocity and temperature fluctuations on the Pitot pressure measurements, the relation between temperature and velocity employed in the above equations will be compared with the experiments of Morkovin and Kistler (refs. (f) and (g)). The temperature fluctuations can be expressed by

$$\frac{T'}{\overline{T}} = -\frac{\gamma-1}{2}\,\overline{\overline{M}}^2\left[(2-\beta)\left(\frac{U'}{\overline{U}}\right) + \left(\frac{U'}{\overline{U}}\right)^2\right] \tag{20}$$

The root-mean-square values of the left- and right-hand side of this equation and neglecting higher order terms provide a relation between temperature and velocity fluctuations.

$$\frac{\sqrt{\overline{T'^2}}}{\overline{T}} = \frac{\gamma-1}{2}\,\overline{\overline{M}}^2(2-\beta)\frac{\sqrt{\overline{U'^2}}}{U} \tag{21}$$

13. Figures 3 and 4 show the measurements of reference (f) of the rms $U'/\overline{U}$ and rms $T'/\overline{T}$, respectively. The line drawn in figure 3 represents the mean of the data. The velocity fluctuation distribution represented by the line was then transformed by equation (21) into temperature fluctuations and plotted in figure 4. The required value of β was computed from the mean profiles as given in the reference. As figure 4 illustrates, equation (21) estimates both the magnitude and the trend of the data. A similar calculation was performed on the data of reference (g) and the resulting calculated points are shown with the measurements in figure 5. In this case each point was transformed into temperature fluctuations. The agreement is again good.

14. The correlation coefficient as calculated from the above equations also compares favorably with the experiment of reference (b), that is:

$$R_{T'U'} = \frac{\overline{T'U'}}{\sqrt{\overline{T'^2}\,\overline{U'^2}}} = -1 \qquad (22)$$

This result was to be expected since similarity in the mechanism of heat and momentum transfer was assumed. The measured correlation coefficient $R_{T'U'}$ (ref. (f)) is shown in figure 6 and is between -.85 and -.90. Kistler in reference (g) also indicates that the correlation coefficient is about -.7.

CALCULATION OF PITOT PRESSURE CORRECTIONS AND DISCUSSION OF RESULTS

15. Basically, equation (19) can be combined with equation (7) and by using the incompressible rms $U'/\bar{U}$ versus y^+ values the error in Pitot pressure due to the fluctuations can be obtained as a function of the local average Mach number and heat transfer. However, because of the number of parameters, it is not possible to give a complete picture of the effect and each situation must be computed separately. Nevertheless, the results can be illustrated by calculating the percentage change in Pitot pressure as a function of local Mach number for different fluctuating velocity levels and by assuming the total temperature fluctuations are zero. This last assumption (i.e., $\beta = 0$) corresponds to zero heat-transfer conditions in the analysis leading to equation (14). The effect of positive β, the most important practical case, is to decrease the error in pressure. Tables 1 and 2 contain the result of such computation based on equations (7) and (19).

16. In order to demonstrate the effect of this analysis as it would be applied, the following example has been calculated. Shown in figure 7 (curve A) are the Pitot pressure ratio measurements in a region extending from the wall to 1/2 mm into a Mach number five turbulent boundary layer (ref. (i)). The pressure tends toward the static pres... at the wall. The solid line drawn through the data represents an arbitrary interpolation for which the corresponding skin-friction coefficient is 12.3 x 10^{-4}. The difference between curve A and B is the correction to the Pitot pressure when equation (19) with $\beta = 0$ and equation (7) are used in connection with figure 2. The solid line through this data corresponds to a skin-friction coefficient of 10.8 x 10^{-4} or a reduction of 12 percent. This calculation assumes negligible total temperature fluctuations. The curve marked C includes the estimated total temperature fluctuations and was computed using equations (7) and (19) and figure 2. The skin-friction coefficient in

this case is 8.5 percent below the original data. The total temperature gradient at the wall divided by the velocity gradient at the wall was used in calculating β.

17. It is necessary to make a remark about the application of the above effects within the laminar sublayer. In that region the percentage velocity fluctuations reach a maximum. However, the fluctuations are generally believed to be basically one dimensional without the cross components that are characteristic of turbulence although this has not been conclusively proven. If, however, it is true, then the convection of fluid normal to the mean flow does not take place and the effects associated with heat transfer are not involved. That is, under this assumption, even with β not zero, the heat-transfer term should not be retained in the laminar sublayer.

REFERENCES

(a) Goldstein, S., "A Note on the Measurement of Total Head and Static Pressure in a Turbulent Stream," Proc. Roy. Soc. Series A, Vol. 155, pp. 570, July 1936

(b) Ribner, H. S., "Convection of a Pattern of Vorticity Through a Shock Wave," NACA TR 1164, 1954

(c) Ribner, H. S., "Shock Turbulence Interaction and the Generation of Noise," NACA TR 1233, 1954

(d) Moore, F. K., "Unsteady Oblique Interaction of a Shock Wave with a Plane Disturbance," NACA TR 1165, 1954

(e) Kistler, A. L., "Fluctuation Measurements in Supersonic Turbulent Boundary Layers," Physics of Fluids 2, pp. 290-296, 3 May - June 1959

(f) Morkovin, M. V. and Phinney, R. E., "Extended Application of Hot-Wire Anemometry to High-Speed Turbulent Boundary Layers," AFOSR-TN-58-469, The Johns Hopkins University, Department of Aeronautics, June 1958

(g) Kistler, A. L., "Fluctuation Measurements in Supersonic Turbulent Boundary Layers," Ballistic Research Laboratories, Report No. 1052, August 1958

(h) Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," NACA Report No. 1247, 1955

(i) Lobb, R. K., Winkler, E. M., and Persh, J., "Experimental Investigation of Turbulent Boundary Layers in Hypersonic Flow," NAVORD Report 3880, March 1955

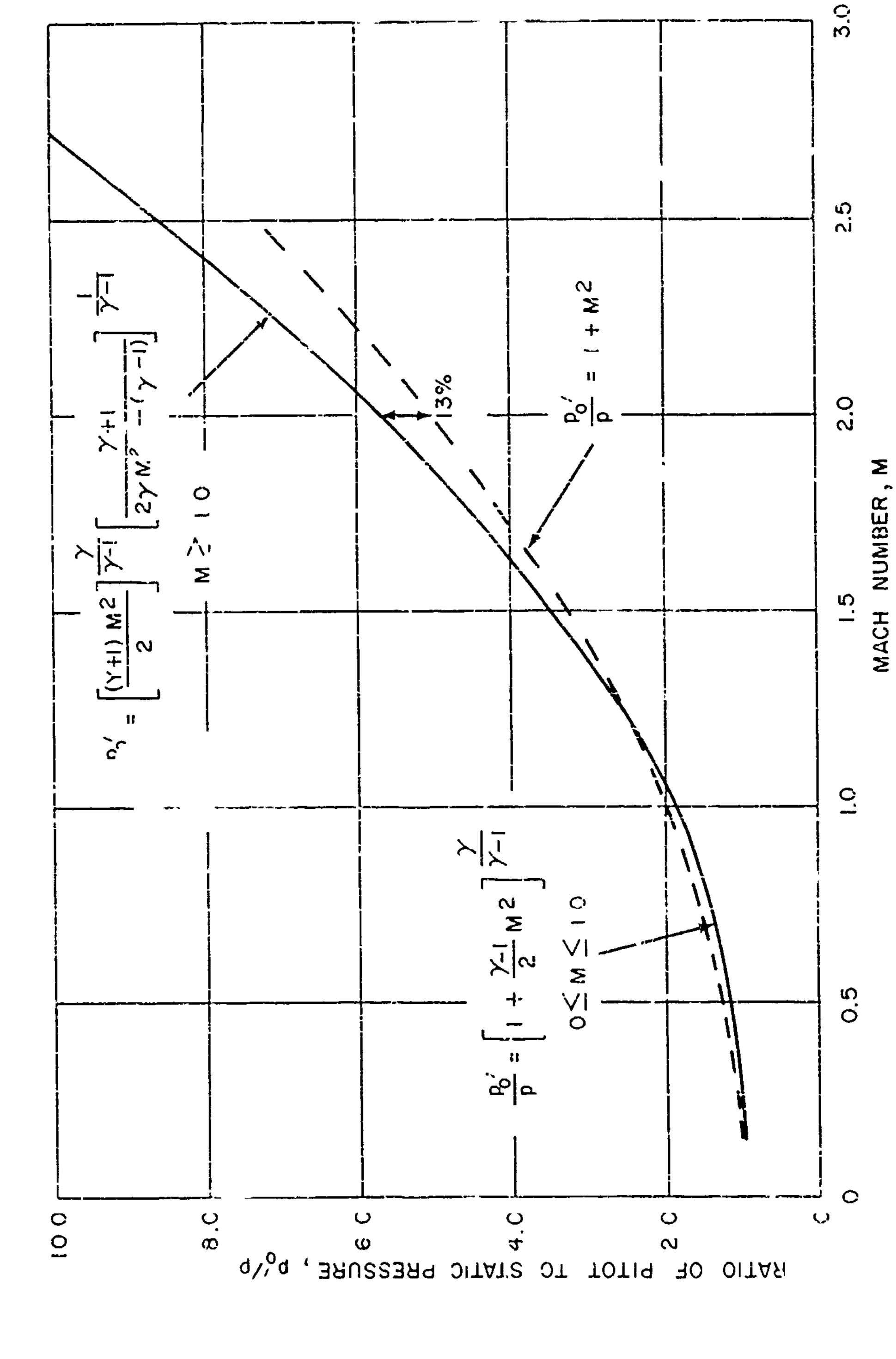

FIG 1 PITOT PRESSURE RATIO VARIATION WITH MACH NUMBER

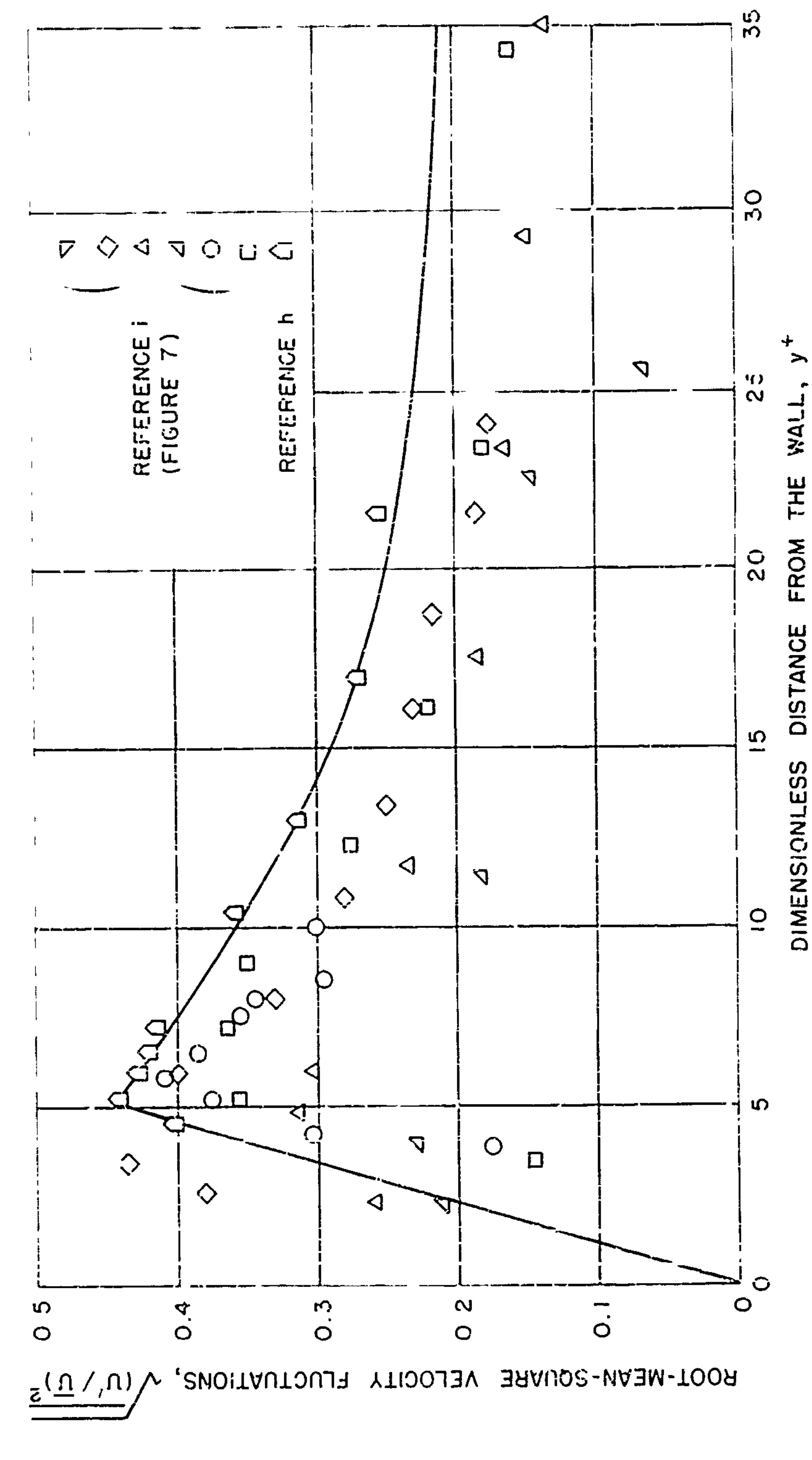

FIG. 2 INCOMPRESSIBLE ROOT-MEAN-SQUARE VELOCITY FLUCTUATIONS

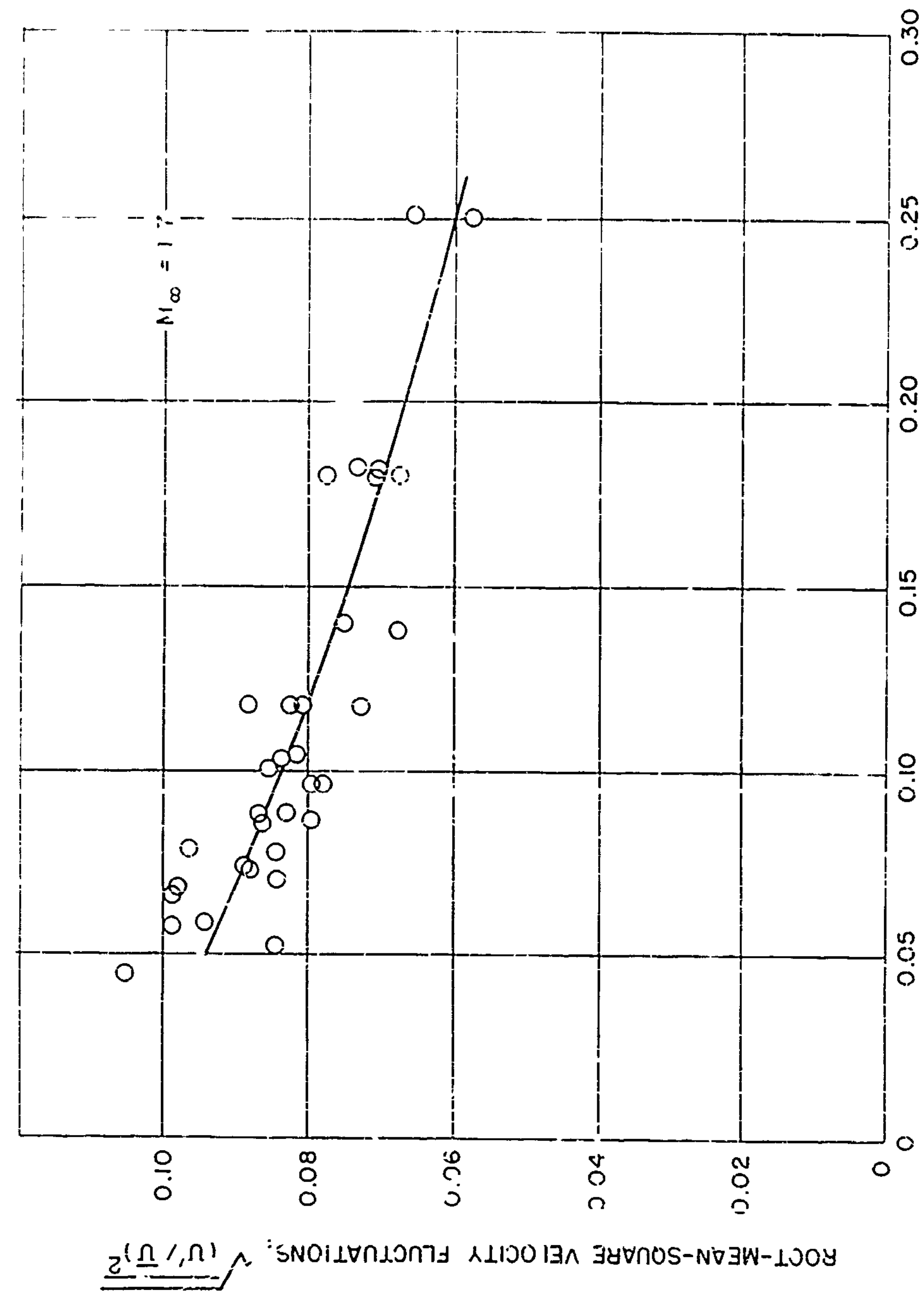

FIG. 3 ROOT-MEAN-SQUARE VELOCITY FLUCTUATIONS (REF. 4)

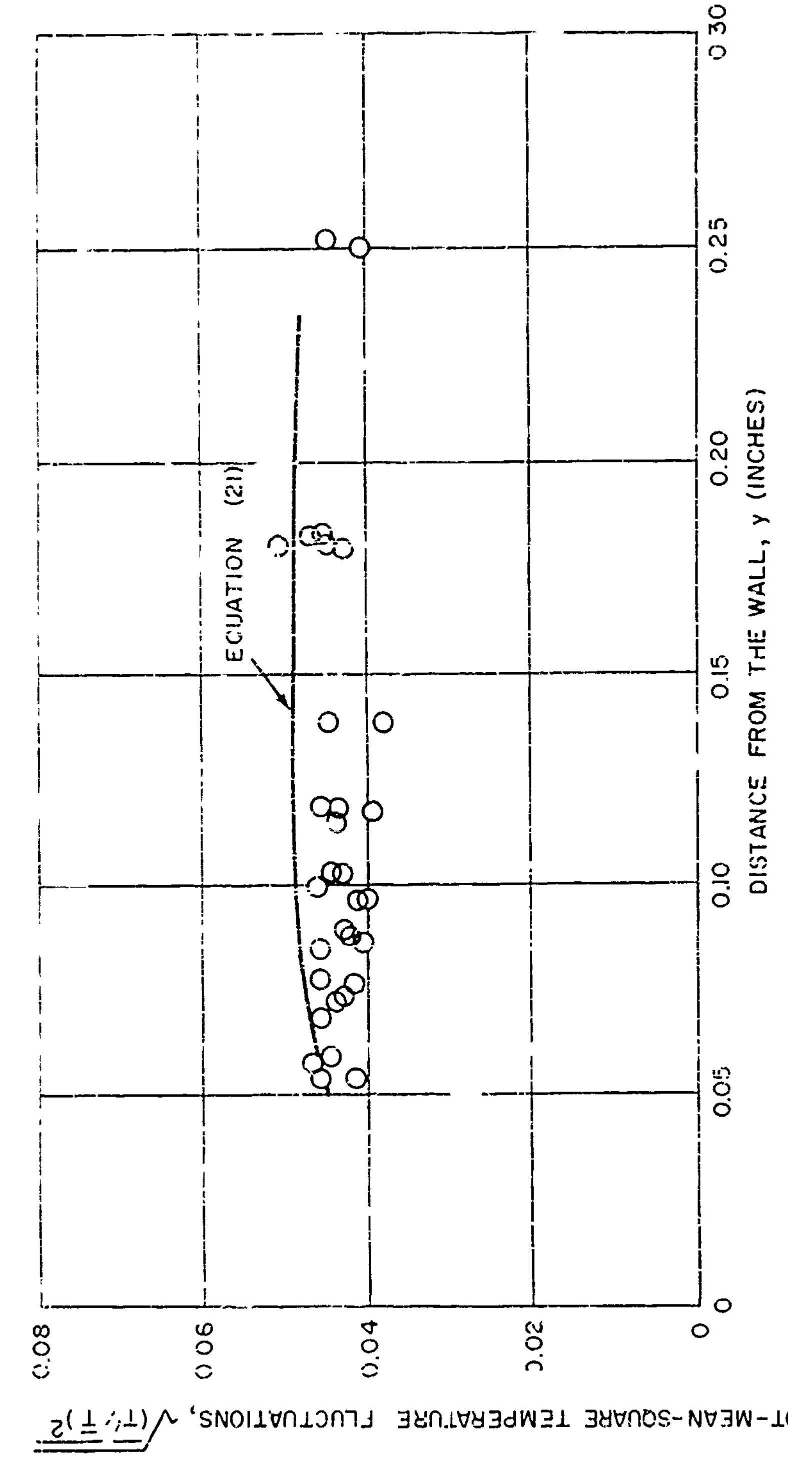

FIG. 4 ROOT-MEAN-SQUARE TEMPERATURE FLUCTUATIONS, (REF f)

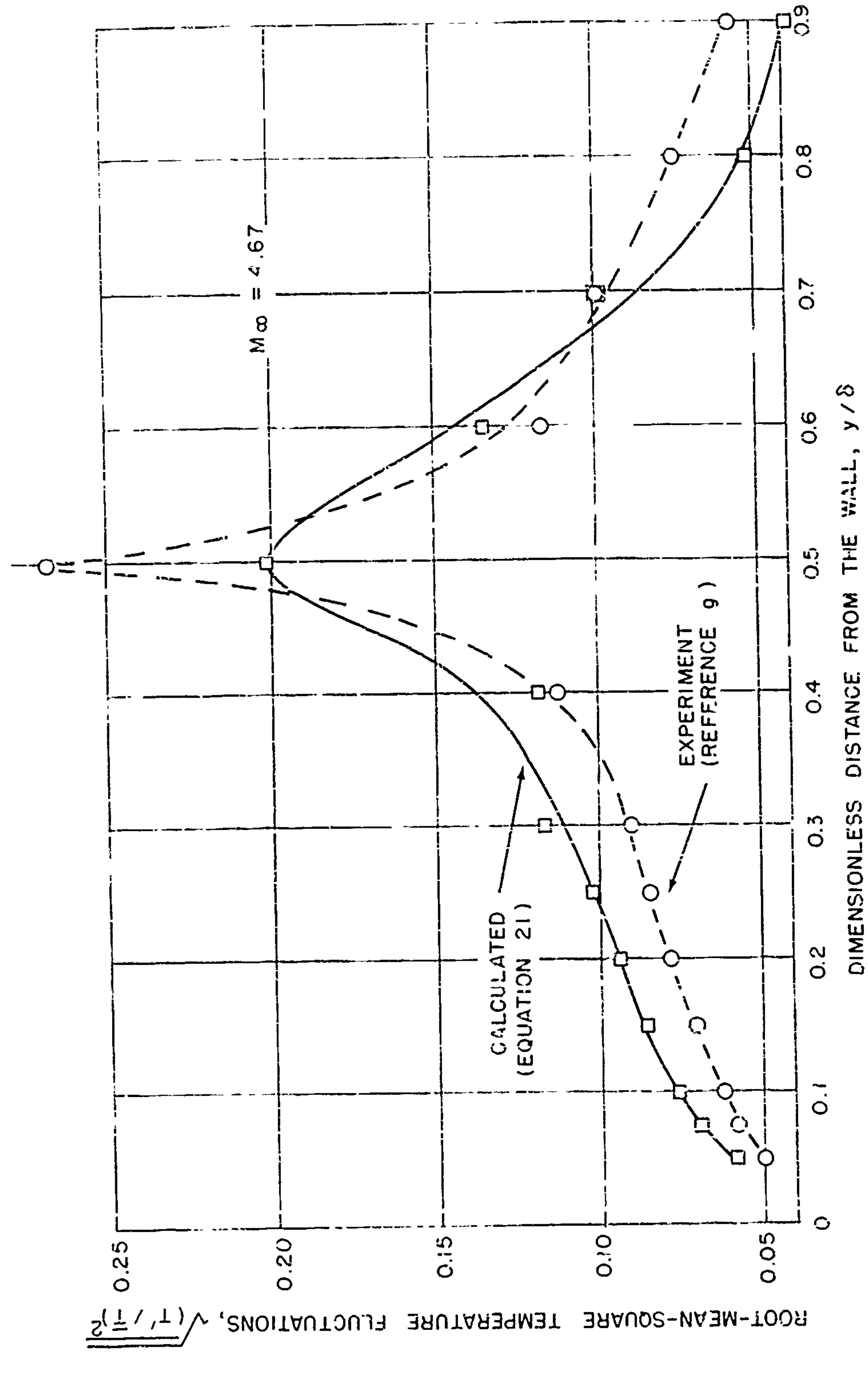

FIG. 5 ROOT-MEAN-SQUARE TEMPERATURE FLUCTUATION (REF g)

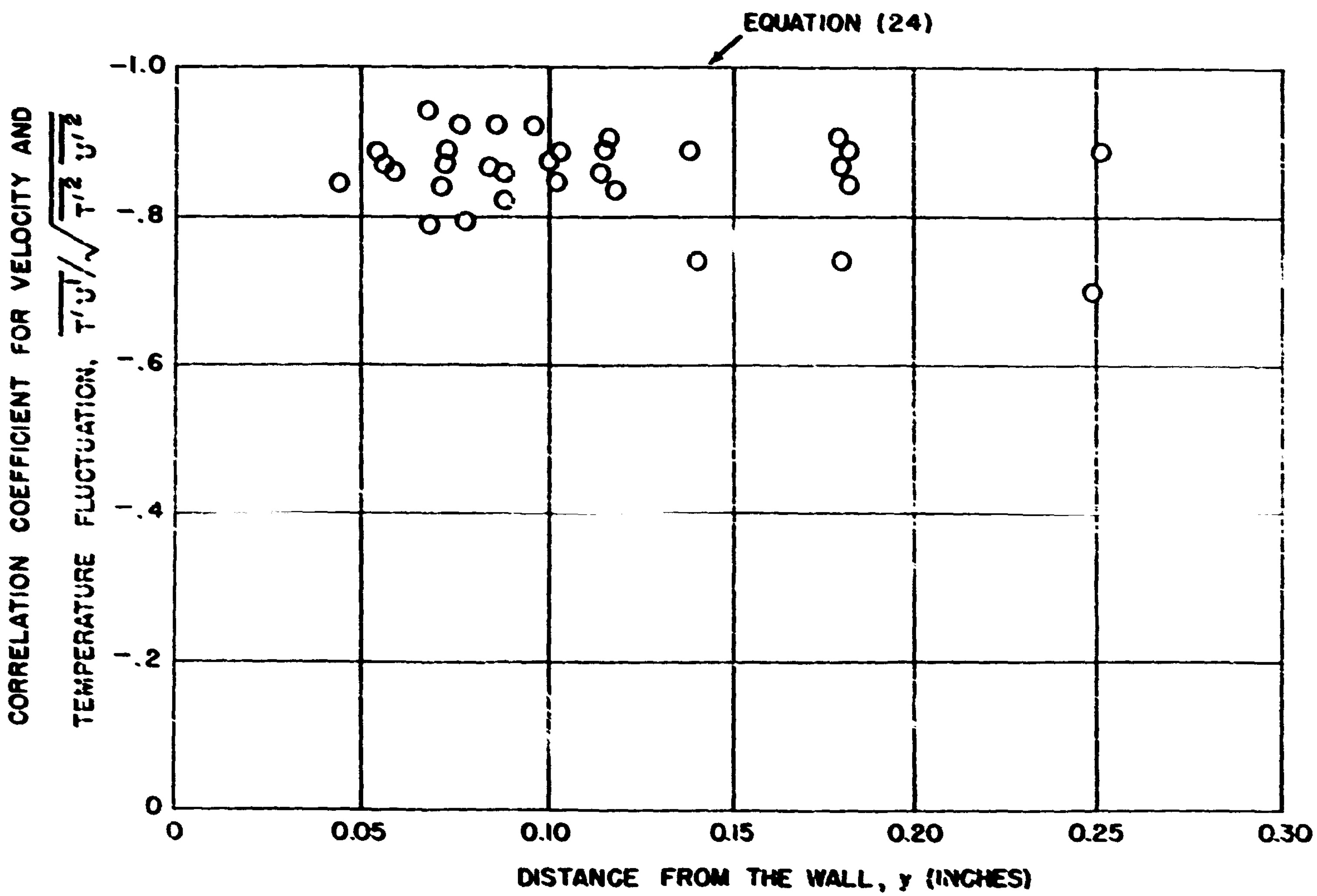

FIG. 6 CORRELATION COEFFICIENT (REF. 1)

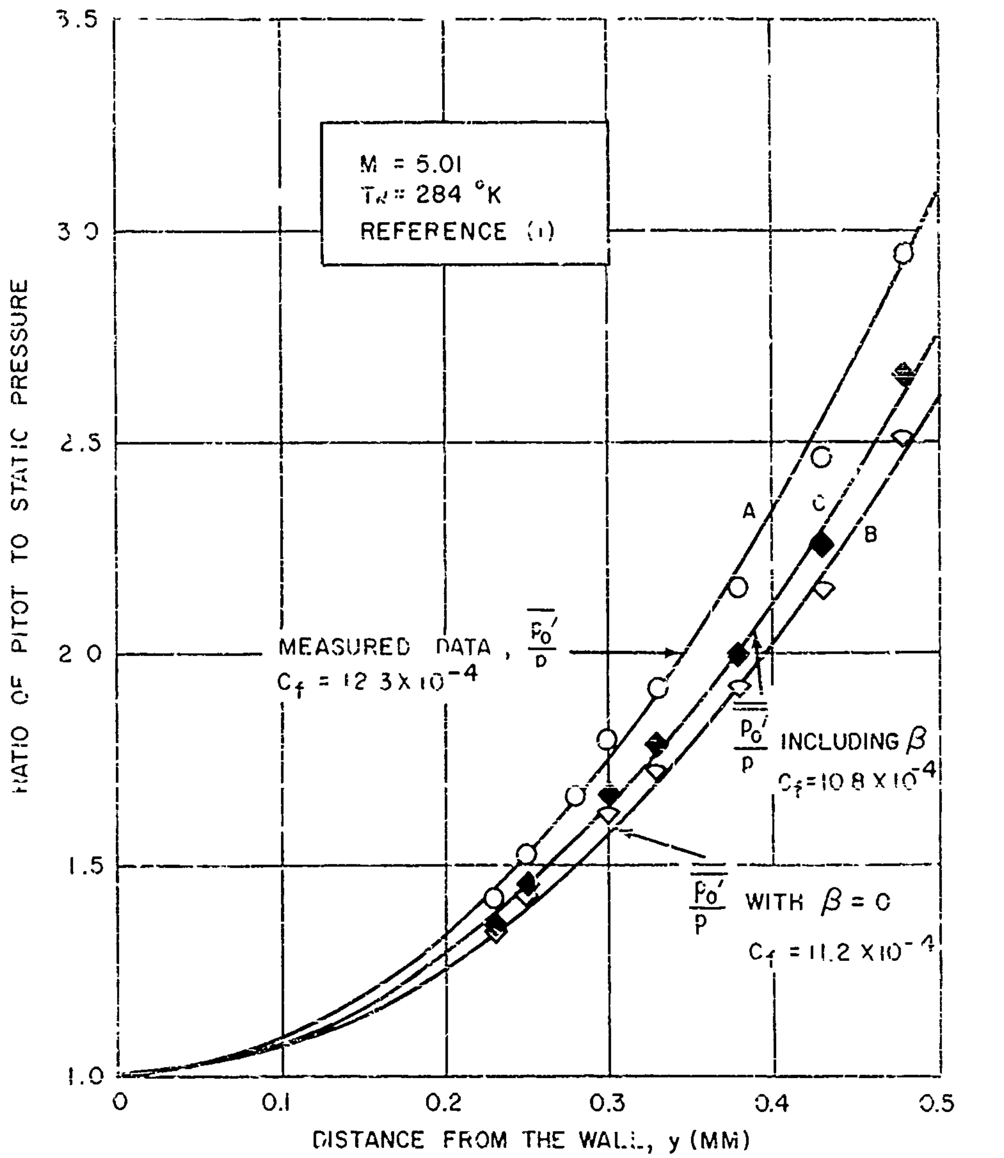

FIG 7 PITOT PRESSURE VARIATION FROM TYPICAL BOUNDARY LAYER SURVEY

Table 1

POSSIBLE ERROR IN PITOT PRESSURE

$\Delta p_o'/\overline{p_o'}$

(Zero Heat Transfer ($\beta = 0$))

$\overline{\overline{M}}$	α	$\overline{(U'/\overline{U})^2}$ .05	.10	.15	.185
0.00	1.00	0.0	0.0	0.0	0.0
0.5	1.26	.0098	.0194	.0288	.0352
1.0	2.16	.0486	.0923	.1323	.1582
1.5	4.06	.1255	.2292		
2.0	7.56	.2372			

$$\frac{\Delta P_o'}{\overline{P_o'}} = \frac{\overline{P_o'} - \overline{\overline{P_o'}}}{\overline{P_o'}} = \left\{ 1 - \frac{1}{1 + \alpha \overline{\left(\frac{U'}{\overline{U}}\right)^2}} \right\} \left(1 - \frac{P}{\overline{P_o'}} \right)$$

$$\alpha = 1 + b + b(2-\beta)(2 + b(2-\beta))$$

IF $\beta = 0$ AND $\gamma = 1.4$

$$\alpha = 1 + \overline{\overline{M}}^2 + \frac{4}{25}\overline{\overline{M}}^4$$

Table 2

ROOT-MEAN-SQUARE MACH NUMBER FLUCTUATIONS $\sqrt{\overline{M'^2}}$

$\bar{\bar{M}}$	$\overline{(U'/\bar{U})^2}$			
	.05	.10	.15	.185
0.0	0.0	0.0	0.0	0.0
0.5	.125	.178	.217	.241
1.0	.3285	.464	.569	.632
1.5	.724	1.024		
2.0	1.229			

$$\sqrt{\overline{M'^2}} \approx \sqrt{\alpha}\sqrt{\overline{\left(\frac{U'}{\bar{\bar{U}}}\right)^2}}\ \bar{\bar{M}}$$

APPENDIX A

A-1. The foregoing analysis developed to estimate the effect of Mach number fluctuations on the Pitot pressure does not apply if the expansion of equation (16) into the infinite series equation (17) does not converge. The following analysis indicates the series does converge in certain ranges of $\overline{\overline{M}}$ and b. These ranges can be calculated as follows.

A-2. Given the infinite series:

$$\frac{M^2}{\overline{\overline{M}}^2} = A_0 + A_1\eta + \cdots A_n\eta^n + \cdots \tag{17}$$

where

$$\begin{aligned} A_0 &= 1 \\ A_1 &= 2 + b(2-\beta) \\ &\vdots \\ A_n &= bA_{n-2} + b(2-\beta)A_{n-1} \\ &\vdots \end{aligned} \tag{18}$$

Such a series converges absolutely if in the limit as n approaches infinity, the absolute value of the ratio of successive terms of the series is less than unity, i.e.,

$$\lim_{n\to\infty}\left|\frac{A_{n+1}}{A_n}\eta\right| < 1 \tag{A-1}$$

From (18):

$$\left|\frac{A_{n+1}}{A_n}\eta\right| = \left|\frac{bA_{n-1} + b(2-\beta)A_n}{b(2-\beta)A_{n-1} + bA_{n-2}}\eta\right| \tag{A-2}$$

Substitute for A_n

$$A_n = bA_{n-2} + b(2-\beta)A_{n-1}$$

$$\left|\frac{A_{n+1}}{A_n}\right|\left|\eta\right| = \left|\frac{[1+b(2-\beta)^2](A_{n-1}/A_{n-2}) + (2-\beta)b}{(2-\beta)(A_{n-1}/A_{n-2}) + 1}\right|\left|\eta\right| \tag{A-3}$$

A-3. A limitation can be placed on the magnitude of η by assuming the Mach number fluctuations are subsonic and do not affect the static pressure. From equation (15)

$$\eta = \frac{U'}{\bar{U}} = \frac{\bar{C}}{\bar{U}}\frac{U'}{\bar{C}} = \frac{M'}{\bar{M}} \approx \frac{M'}{\bar{\bar{M}}} \tag{15}$$

Where $M' < 1$, the maximum value that η can have is

$$|\eta| \leq 1/\bar{\bar{M}} \tag{A-4}$$

Thus, the series converges when the maximum value of the coefficient ratio is:

$$\lim_{n \to \infty} \left| \frac{A_{n+1}}{A_n} \right| \leq \frac{1}{|\eta|} \approx \bar{\bar{M}} \tag{A-5}$$

In the limit as $n \to \infty$, the following also holds

$$\lim_{n \to \infty} \left| \frac{A_{n-1}}{A_{n-2}} \right| \leq \frac{1}{|\eta|} \approx \bar{\bar{M}} = \sqrt{\frac{2b}{\gamma - 1}} \tag{A-6}$$

Substitution of equations (A-5) and (A-6) into equation (A-3) results in an equation governing the maximum values of b and β such that

$$1 = \left| \frac{\left[1 + b_m(2-\beta)^2\right]\bar{\bar{M}}_m + b_m(2-\beta)}{(2-\beta)\bar{\bar{M}}_m + 1} \right| \frac{1}{\bar{\bar{M}}_m} \tag{A-7}$$

If consideration is limited to values of $\beta_m < 2$ every term in equation (A-7) is positive and the absolute sign can be dropped. After rearrangement the condition on $\bar{M}$ and β in order that the series converge absolutely is

$$\bar{\bar{M}}_m = \frac{3 - \gamma}{(\gamma - 1)(2 - \beta_m)} \tag{A-8}$$

or if $\gamma = 1.4$

$$\bar{\bar{M}}_m = \frac{4}{2 - \beta_m} \tag{A-9}$$

A-4. The results are summarized in the following table.

	β_{1a}	M_m
Heat transfer from the wall	-1.0	1.33
	-0.5	1.60
Adiabatic	-0.0	2.00
Heat transfer to the wall	0.1	2.10
	0.2	2.22
	0.5	2.67
	1.0	4.00
	1.5	8.00
	2.0	∞

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2. Pitot tubes - Pressure distribution
3. Boundary layer - Temperature
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